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Navier-Stokes-like equations applicable to adaptive cruise control traffic flows

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Abstract. Under the scenario in which, within a traffic flow, each vehicle is controlled by adaptive cruise control (ACC), and the macroscopic one-vehicle probability distribution function fits the Paveri-Fontana hypothesis, a set of reduced Paveri-Fontana equations considering the ACC effect is derived. With the set, by maximizing the specially defined informational entropy deviating from a certain reference homogeneous steady state, the Navier-Stokes-like equations considering ACC are introduced. For a homogeneous steady traffic flow in a single circular lane, when the steady velocity or density is perturbed along the lane, numerical simulations indicate that ACC-controlled vehicles require less time for re-equilibration than manually driven vehicles. The re-equilibrated steady densities for ACC and manually driven traffic flows are all close to the original values; the same is true for the re-equilibrated steady velocity for manually driven traffic flows. For ACC traffic flows, the re-equilibrated steady velocity may be higher or lower than the original value, depending upon a parameter ω (introduced to solve the distribution function of the reference steady state), and the headway time (introduced in ACC models). Also, the simulations indicate that only an appropriate parameter set can ensure the performance of ACC; otherwise, ACC may result in low traffic running efficiency, although traffic flow stability becomes better.

PACS. 45.70.Vn Granular models of complex systems; traffic flow – 02.50.-r Probability theory, stochastic processes, and statistics

1 Introduction

As traffic conditions drastically deteriorate, the issues regarding traffic flows have attracted the enthusiasm of many physicis and engineers. Traffic models play an important role within today's traffic research. They have been successfully used in many traffic applications, such as traffic flow prediction, incident detection, and traffic control [2]. Based on the level of detail, traffic flow models are categorized into microscopic, mesoscopic, and macroscopic models [8,14]. The models of different detail levels are not isolated; links exist between them [1,7,11]. For instance, a connection between a microscopic follow-the-leader model and a semi-discretization of a macroscopic continuum model has been established, based on a conservation law [1]. Also, a macroscopic continuum model has been obtained from microscopic car following models [6].

These traffic flow models are founded upon different backgrounds, such as the Maxwell model [15] or gas-kinetic theory [7,23,24]. Some are based on cellular automata, which often take real-time traffic information (provided by modern intelligent transportation systems) [13,26] into account. The many other approaches include: all kinds of car-following models [10,12,13,21,22], optimal velocity models [3,16], the continuum traffic model [25], the full velocity and acceleration difference model [27] and some new models [17,19]. With the development of traffic dynamic analysis, some properties, such as stability [5] and bifurcation [9,18] have also been studied.

Of all these models, the set of Navier-Stokes-like equations, coming from fluid- and gas- kinetic analysis [5,7,8], is the appropriate for the study of traffic flows. In this model, a single vehicle is seen as a gas particle, the Boltzman-like term is used to describe the interactions among vehicles macrocosmically, and the time derivative of the instantaneous velocity is assumed to be the quotient of the difference between the instantaneous velocity and the desired velocity and a constant relaxation time [24], or a function of vehicle density [7]. However, in practice, contrary to gas dynamics, there are not enough cars on a road to justify a Boltzmann approximation, and cars that only have positive velocities are different from the particles in gas dynamics. Furthermore, the role of diffusion is not clear in traffic flows.

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For adaptive cruise control traffic flows, the strategy regulating instantaneous velocity is controlled by ACC; the assumption of relaxing the instantaneous velocity with a constant time [7,24] is not appropriate. In a more realistic scenario, the relaxation of the instantaneous velocity is not only dependent upon the difference between the desired velocity and the instantaneous velocity, but also on the difference between the instantaneous velocity of a vehicle and that of the one preceding it. Furthermore, the desired velocity is variable, and relies on the vehicle density values in the vicinity of the considered vehicle. Taking these considerations and the macroscopic traffic flow model given in [24] into account, this paper derives a set of improved Navier-Stokes-like equations applicable to ACC traffic flows.

The rest of this paper is organized as follows: in Section 2, a new expression of ACC dynamics is derived. Section 3 is devoted to constructing the Paveri-Fontana equation considering ACC. The improved Navier-Stokes equations are derived in Section 4. Some numerical simulations and discussions, and concluding remarks, are given in Sections 5 and 6, respectively.

2 A new expression of ACC dynamics

The aim of ACC is to maintain the headway between a vehicle and the one that is preceding it [4]. Modern technologies provide many ways to obtain the headway accurately, such as radar, image sequence processing, etc. The application of the obtained headway into the relaxation of instantaneous velocity can improve the stability of the traffic flow by eliminating dangerous interactions among vehicles. The kernel of ACC is the algorithm controlling the headway. Usually, the constant-headway time policy is favorable for its stable quality [4].

In a queue of N vehicles, where the head one is numbered $n = 0$, and the tail one is $n = N - 1$, the adaptive cruise control system is described by [4]:

$$\tau \frac{d\nu_n(t)}{dt} + \nu_n(t) = \frac{1}{h_d}(\Delta x_n(t) - l_n) + \beta \Delta\nu_n(t), \tag{1}$$

where τ models the vehicle response time, which is dependent upon the vehicle performance and the ACC system etc. (τ is typically 0.5–1.0 s). $\nu_n(t)$ is the instantaneous velocity of the nth vehicle. h_d is the headway time, for a constant-time headway policy, it is a constant, generally about 1 s. Let

$$\Delta x_n(t) = x_{n-1}(t) - x_n(t) \quad \text{and} \quad \Delta\nu_n(t) = \nu_{n-1} - \nu_n(t) \tag{2}$$

be the distance and velocity differences between a vehicle and that preceding it. l_n is the length of the nth vehicle. β is often a constant, dependent upon τ and h_d. The desired division between the $(n-1)$th vehicle and the nth vehicle is given by $\Delta x_n^d(t) = h_d\nu_n(t) + l_n$, so the desired velocity for vehicle n can be taken as follows

$$w_n(t) = \frac{1}{h_d}[\Delta x_n(t) - l_n]. \tag{3}$$

In practice, in a single unidirectional lane, the vehicle position is completely described by x. At time t, assume the nth vehicle is at x. Let $h(x,t)$, $\nu(x,t)$, and $w(x,t)$ respectively denote the sum of the headway and the vehicle length l_n, $\nu_n(t)$, and $w_n(t)$. Under these circumstances, equation (1) can be written as

$$\tau \frac{d\nu(x,t)}{dt} + \nu(x,t) = \frac{1}{h_d}(h(x,t) - l(x,t)) + \beta[\nu(x - h(x,t),t) - \nu(x,t)]. \tag{4}$$

From position $x - h(x,t)$ to x, there exists only one vehicle, so

$$\rho(x,t) = 1/h(x,t) \tag{5}$$

is the vehicle density at position x and time t. Write (3) into

$$w(x,t) = (h(x,t) - l(x,t))/h_d. \tag{6}$$

Actually $[\nu(x - h(x,t),t) - \nu(x,t)]/h(x,t)$ means the difference quotient of the instantaneous velocity along x. Let

$$\frac{\partial}{\partial x}\nu(x,t) = \frac{\nu(x - h(x,t),t) - \nu(x,t)}{h(x,t)}. \tag{7}$$

From equations (4)–(7), we have

$$\rho(x,t)\frac{d\nu(x,t)}{dt} = \frac{\rho(x,t)[w(x,t) - \nu(x,t)]}{\tau} + \frac{\beta}{\tau}\frac{\partial\nu(x,t)}{\partial x}. \tag{8}$$

Equation (8) is the reformed kernel control algorithm of ACC, and has typical realistic meaning: Usually, when the instantaneous velocity exceeds the desired velocity, that is when $\nu(x,t) - w(x,t) > 0$, the driver will slow the vehicle down. This factor is involved in (8) as $\rho(x,t)[w(x,t) - \nu(x,t)] < 0$ reduces $\rho(x,t)\partial\nu(x,t)/\partial t$. On the other hand, $\partial\nu(x,t)/\partial x > 0$ implies that the preceding vehicle has a higher velocity. In this case, the vehicle should speed up. Equation (8) clearly confirms this. Equation (3) in reference [24] shows the following relaxation policy

$$\partial\nu(x,t)/\partial t = [w(x,t) - \nu(x,t)]/\mp \quad \text{and} \quad \partial w(x,t)/\partial t = 0, \tag{9}$$

where $\mp$ is a fixed relaxation time. Compared with equation (9), equation (8) seems to be more compatible with realistic cases, as it considers the instantaneous velocity difference between the vehicle and that preceding it, and that in different cases the desired velocity changes.

3 The Paveri-Fontana equations considering ACC Dynamics

For a single unidirectional vehicle lane, the Paveri-Fontana model [20] abstracts the traffic state by the one-vehicle distribution function $g(x,c,b,t)$, such that $g(x,c,b,t)dxdcdb$ is the count of the vehicles at time t, in the road interval between x and $x + dx$ and their instantaneous and desired velocities are between c and $c + dc$, and b and $b + db$, respectively. The function $g(x,c,b,t)$ satisfies the following gas-kinetic traffic equation [24]

$$\frac{\partial g(x,c,b,t)}{\partial t} + c\frac{\partial g(x,c,b,t)}{\partial x} + \frac{\partial}{\partial c}\left(g(x,c,b,t)\frac{dc}{dt}\right) + \frac{\partial}{\partial b}\left(g(x,c,b,t)\frac{db}{dt}\right) = f(x,c,t) \int_c^\infty (1-p)(c'-c)g(x,c',b,t)dc' - g(x,c,b,t)\int_0^c (1-p)(c-c')f(x,c',t)dc' \quad (10)$$

where p is the probability that a slower vehicle can be immediately overtaken, $f(x,c,t) = \int_0^\infty g(x,c,b,t)db$ is the one-vehicle instantaneous velocity distribution function at position x and time t. The right-hand side of equation (10) is the macroscopical collision term, derived from some previously described assumptions [7,24]. The main shortcoming of the Paveri-Fontana model is that it is very difficult to obtain the analytical solution when the interaction process can not be neglected. To overcome this shortage, under the assumptions

$$\lim_{b\to 0} g(x,c,b,t) = 0 \quad \text{and} \quad \lim_{b\to\infty} g(x,c,b,t) = 0, \quad (11)$$

a reduced Paveri-Fontana equation (12) is obtained by integrating the two sides of equation (10) with the desired velocity b

$$\frac{\partial f(x,c,t)}{\partial t} + c\frac{\partial f(x,c,t)}{\partial x} + \frac{\partial}{\partial c}\left(\int_0^\infty g(x,c,b,t)\frac{dc}{dt}db\right) = f(x,c,t)\int_0^\infty (1-p)(c'-c)f(x,c',t)dc'. \quad (12)$$

Assuming the macroscopic statistic distribution of the traffic follow constituted by ACC vehicles fits $g(x,\nu,w,t)$, by equation (8) we take

$$\frac{dc(x,t)}{dt} = \frac{b(x,t)-c(x,t)}{\tau} + \frac{\beta}{\tau p(x,t)}\frac{\partial c(x,t)}{\partial x}, \quad (13)$$

which is an assumption imposed on model (12), as the vehicles are presumed to be microscopically controlled by ACC. Insertion of the above equation into (12) yields

$$\frac{\partial f(x,c,t)}{\partial t} + c\frac{\partial f(x,c,t)}{\partial x} + \frac{\partial}{\partial c}\left(\frac{\bar{b}(x,c,t)-c}{\tau}f(x,c,t)\right) + \frac{\partial}{\partial c}\left(f(x,c,t)\frac{\beta}{\tau p(x,t)}\frac{\partial c}{\partial x}\right) = f(x,c,t)\int_0^\infty (1-p)(c'-c)f(x,c',t)dc' \quad (14)$$

where

$$\bar{b}(x,c,t) = \int_0^\infty b\frac{g(x,c,b,t)}{f(x,c,t)}db$$

is the average desired velocity for ACC vehicles passing x, at time t, with an instantaneous velocity c. Equation (14) is the reduced Paveri-Fontana equation for the traffic flow considering ACC.

4 The improved Navier-Stokes-like equations

Assuming

$$\lim_{c\to 0} f(x,c,t) = 0 \quad \text{and} \quad \lim_{c\to\infty} f(x,c,t) = 0, \quad (15)$$

denoting

$$\bar{c}(x,t) = \int_0^\infty c\frac{f(x,c,t)}{p(x,t)}dc,\ \bar{b}(x,t) = \int_0^\infty b(x,c,t)\frac{f(x,c,t)}{p(x,t)}dc,$$

and noticing that

$$\rho(x,t) = \int_0^\infty (x,c,t)dc, \quad (16)$$

integration of equation (14) with c from 0 to ∞ yields the continuity equation.

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial}{\partial x}(\rho(x,t)\bar{c}(x,t)) = 0. \quad (17)$$

Using (15) and (16), with the two sides of equation (14) multiplying and integrating with c, it follows that

$$\rho(x,t)\left(\frac{\partial\bar{c}(x,t)}{\partial t} + \bar{c}(x,t)\frac{\partial\bar{c}(x,t)}{\partial x}\right) + \frac{\partial}{\partial x}p(x,t) = \frac{\rho(x,t)}{\tau}\left[\bar{b}(x,t)-\bar{c}(x,t)\right] + \frac{\beta}{\tau}\frac{\partial}{\partial x}\bar{c}(x,t) - (1-p)\rho(x,t)p(x,t), \quad (18)$$

where

$$p(x,t) = \int_0^\infty (c-\bar{c}(x,t))^2 f(x,c,t)d\nu = \rho(x,t)\Theta(x,t) \quad (19)$$

is the traffic pressure, $\Theta(x,t)$ denotes the variance of the instantaneous velocity c. By the Chapman-Enskog method, the distribution function $f(x,c,t)$ can be written as

$$f(x,c,t) = f^0(x,c,t) + f^1(x,c,t) + \ldots \quad (20)$$

where $f^i(x,c,t)$ denotes the successive approximations for the distribution function. As has been done in previous literature [24], we assume the distribution functions corresponding to a homogeneous steady state not to be dependent upon time t and position x, and denote the macroscopic quantities corresponding to this state as

$$\rho_e = \int_0^\infty f_e(c)dc, \qquad \rho_e c_e = \int_0^\infty c f_e(c)dc. \quad (21)$$

With this steady state, as $f(x,c,t)$ is assumed to be independent of x and t, becomes $f_e(c)$, leading to $\frac{\partial}{\partial t}f_e(c) = \frac{\partial}{\partial x}f_e(c) = 0$, the reduced Paveri-Fontana model (14) becomes

$$\frac{\partial}{\partial c}\left(f_e(c)\frac{\bar{b}(c)-c}{\tau}\right) = f_e(c)\rho_e(1-p)(c_e-c). \quad (22)$$

Based upon equation (6), the following can be written

$$\bar{b}(c) = \frac{1}{h_d}\left(\frac{1}{\rho_e} - D\right), \tag{23}$$

where D is a constant length that is a little longer than the maximal vehicle length (often 7 m) [4]. This gives

$$\frac{\partial}{\partial c}\left(f_e(c)\frac{1-\rho_e D - \rho_e h_d c}{\rho_e h_d \tau}\right) = f_e(c)\rho_e(1-p)(c_e - c). \tag{24}$$

In equation (24), $f_e(c)$ can be written as

$$f_e(c) = \frac{\alpha}{\Gamma(x)}\frac{\rho_e}{c_e}\left(\frac{\alpha c}{c_e}\right)^{\alpha-1}\exp\left(-\frac{\alpha c}{c_e}\right),$$
$$\alpha = \frac{\tau\rho_e(1-p)c_e}{\omega/h_d - 1} \tag{25}$$

under the assumption

$$1 - \rho_e D = \rho_e \omega c, \quad \omega > h_d, \tag{26}$$

where α is a dimensionless constant characterizing the homogeneous steady state. Equation (26) is equivalent to $h_e - D = \omega c$, $h_e = 1/\rho_e$, is the headway within the reference homogeneous steady state, ω is the time the vehicle needs to go through the headway $h_e - D$ with velocity c. $\omega > h_d$ means ρ_e is relatively small. In a traffic jam, the actual velocity is small, and a very large ω can satisfy equation (26). Even equation (26) will fail when $c = 0$. It is worth noting that equation (26) can be replaced by others, it is not unique but is mathematically sound.

In order to calculate $f^1(x,c,t)$, the mean free interaction time τ_0 is needed [7,24]. When taking $f(x,c,t) = f^0(x,c,t) + f^1(x,c,t)$, the traffic pressure is calculated as

$$P = \frac{\rho(x,t)\bar{c}^2(x,t)}{\alpha}\left(1 - \tau^*\frac{\partial \bar{c}(x,t)}{\partial x}\right), \tag{27}$$
$$\tau^* = 2\tau_0(1+\alpha)/\alpha.$$

Insertion of equation (27) into equation (18) gives

$$\begin{aligned}\frac{\partial \bar{c}(x,t)}{\partial t} =& -\frac{\bar{c}^2(x,t)}{\alpha\rho(x,t)}\left[\frac{2\rho(x,t)}{\bar{c}(x,t)}\frac{\partial \bar{c}(x,t)}{\partial x} + \frac{\partial \rho(x,t)}{\partial x}\right.\\ &\left. + (1-p)\rho^2(x,t)\right]\left(1 - \tau^*\frac{\partial \bar{c}(x,t)}{\partial x}\right)\\ &+ \frac{\bar{c}^2(x,t)}{\alpha}\tau^*\frac{\partial^2 \bar{c}(x,t)}{\partial x^2} + \frac{\bar{b}(x,t) - \bar{c}(x,t)}{\tau}\\ &+ \frac{\beta}{\tau\rho(x,t)}\frac{\partial}{\partial x}\bar{c}(x,t) - \bar{c}(x,t)\frac{\partial \bar{c}(x,t)}{\partial x}.\end{aligned} \tag{28}$$

Equations (17) and (28) constitute the new Navier-Stokes-like equation set considering the adaptive cruise control effect. Compared to the ordinary Navier-Stokes-like equation set (Eqs. (9) and (37) in [24]), $\frac{\beta}{\tau\rho(x,t)}\frac{\partial}{\partial x}\bar{c}(x,t)$ is a new term resulting from the ACC effect.

5 Numerical simulations and discussions

From [24], the probability of passing takes the explicit form

$$p = 1 - \frac{\rho}{\hat{\rho}}, \tag{29}$$

where $\hat{\rho}$ is the maximum vehicular density. There are many ways to specify β. Using

$$\beta = \frac{\tau}{h_d} \tag{30}$$

eliminates the difference between $c(x,t)$ and $b(x,t)$ with time evolution [4]. Let the desired velocity be controlled by ACC

$$\bar{b}(x,t) = \frac{1}{h_d}\left[\frac{1}{\rho(x,t)} - D\right]. \tag{31}$$

Insertion of equations (27), (29)−(31) into equation (28) gives

$$\begin{aligned}\frac{\partial \bar{c}(x,t)}{\partial t} =& \frac{2\tau_0(1+\alpha)\bar{c}^2(x,t)}{\alpha^2}\frac{\partial^2 \bar{c}(x,t)}{\partial x^2} - \frac{\bar{c}^2(x,t)}{\alpha}\\ &\times\left[\frac{2}{\bar{c}(x,t)}\frac{\partial \bar{c}(x,t)}{\partial x} + \frac{1}{\rho(x,t)}\frac{\partial \rho(x,t)}{\partial x} + \frac{\rho^2(x,t)}{\hat{\rho}}\right]\\ &\times\left(1 - 2\tau_0\frac{1+\alpha}{\alpha}\frac{\partial \bar{c}(x,t)}{\partial x}\right) \qquad (32)\\ &+ \frac{1}{\tau}\left[\frac{1}{h_d}\left(\frac{1}{\rho(x,t)} - D\right) - \bar{c}(x,t)\right]\\ &+ \frac{1}{h_d\rho(x,t)}\frac{\partial \bar{c}(x,t)}{\partial x} - \bar{c}(x,t)\frac{\partial \bar{c}(x,t)}{\partial x}\end{aligned} \tag{33}$$

where

$$\alpha = \frac{h_d\tau\rho_e^2 c_e}{(\omega - h_d)\hat{\rho}} \tag{34}$$

resulted from equations (25) and (29). From equation (33), once the headway time h_d, response time τ of ACC vehicles, ρ_e, c_e, $\hat{\rho}$ and are given, we know α and ω are interdependent.

Let $L = 15$ km, $\hat{\rho} = 140$ vehicles/ km, D =7.0 m, and consider a homogeneous steady traffic flow described by $\rho_e = 35$ vehicles/km, $c_e = 75$ km/h. The pending parameters about the ordinary Navier-Stokes-like equations [24] are given as $\alpha = 100$ and the mean interaction time $\tau_0 = 300$ s. The values in our model are vehicle response time $\tau = 0.6$ s, ACC headway time $h_d = 1$ s, and the parameters introduced in (26) $\omega = 1.5$ s and in (27) $\tau_0 = 60$ s (commonly, the interaction time of an ACC traffic flow is less than that of the traffic flow without ACC). The initial and boundary conditions are uniformly provided as

$$\begin{aligned}&\rho(x,0) = \rho_e, \qquad V(x,0) = V_e + 5\sin(2\pi x/L),\\ &\rho(0,t) = \rho(L,t), \qquad V(0,t) = V(L,t),\\ &\frac{\partial}{\partial x}V(0,t) = \frac{\partial}{\partial x}V(L,t).\end{aligned} \tag{35}$$

The simulated results of the model given in [24] are shown in Figures 1, 2, 5, and 6, and the results of our model

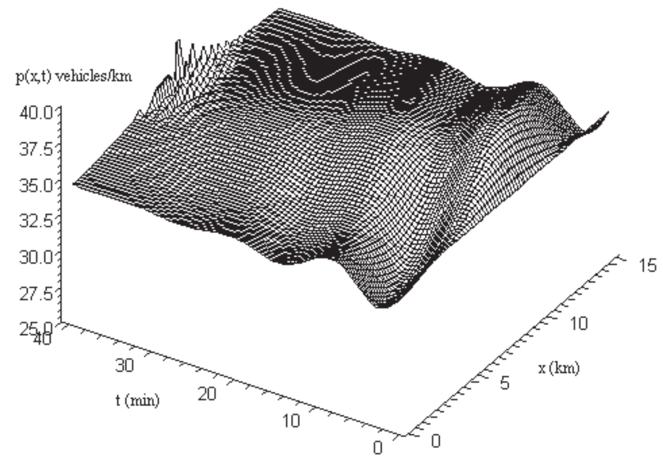

Fig. 1. Vehicular density controlled by the model given in [1].

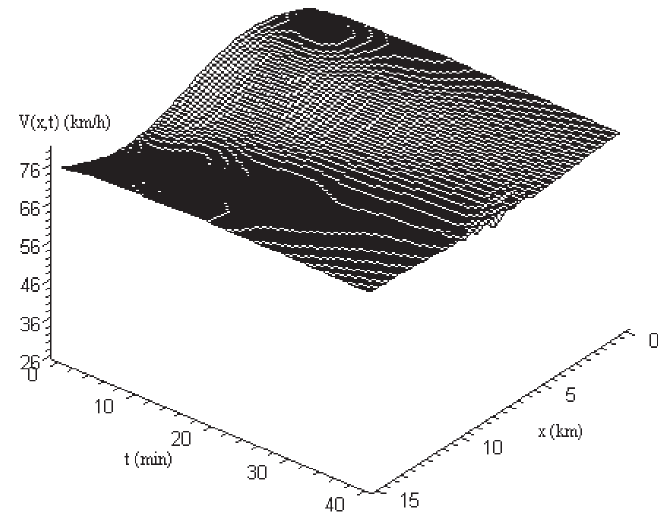

Fig. 2. Average velocity controlled by the model given in [1].

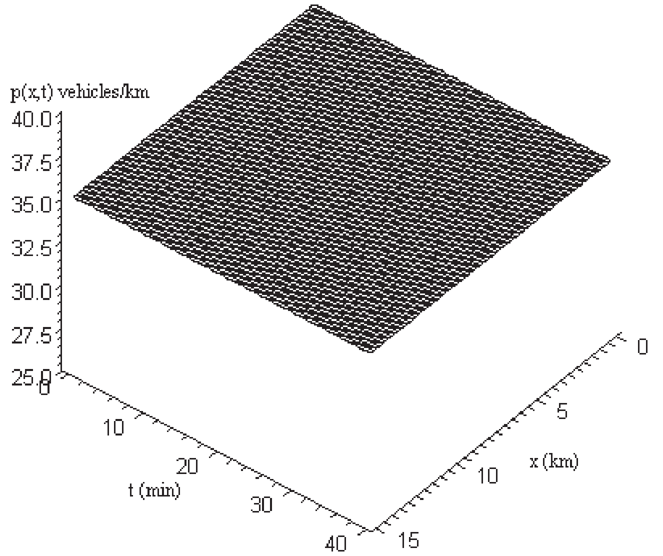

Fig. 3. The vehicular density of our model.

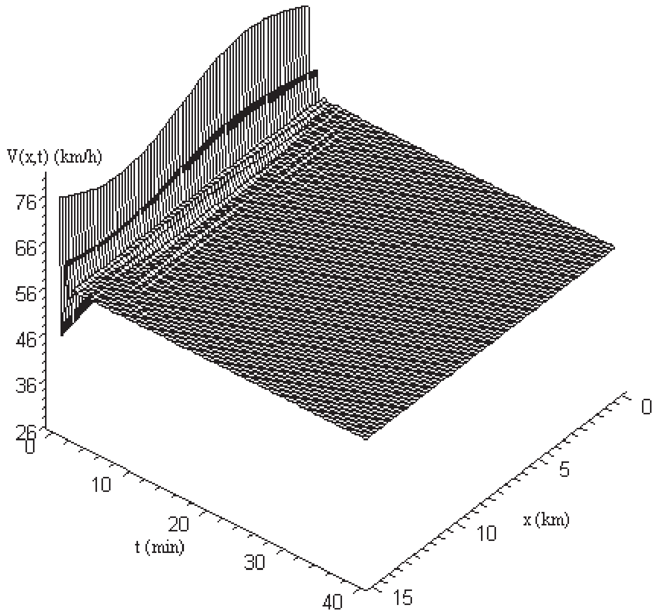

Fig. 4. The average velocity of our model.

are shown in Figures 3, 4, 7, and 8. The comparison of Figure 1 with 3 and 2 with 4 show that the perturbed ACC traffic flow has a higher speed in entering into a steady state. Compared with the original steady state, the re-equilibrated steady velocity of ACC traffic flow is about 56 km/h (Fig. 7), which is lower than that of the manually driven traffic flow (which is near to 75 km/h) (Fig. 5). The re-equilibrated steady vehicular densities of the two models are all equal to the original values (35 vehicles/km) (Figs. 6 and 8).

Let u_e denote the re-equilibrated steady velocity of the perturbed ACC traffic flow. To investigate how ω and h_d affect u_e, some theoretical numerical simulations are performed, the results of which are shown in Figure 9. It can be seen that as ω increases, u_e decreases. The increase of ω will increase the headway at which a vehicle (whose velocity is relatively large) begins to decelerate (resulting from the interaction of it and the preceding vehicle), finally causing the perturbed ACC traffic flow to develop into a steady state with low velocity. Also, u_e may be higher than the original value c_e when ω is small enough and satisfies $\omega > h_d$, as required by (26). For instance, when $\omega = 1.01$, $\tau = 0.6$, $h_d = 1.0$ or 0.8, then $u_e > c_e$. From Figure 9, it can be seen that with the same ω and τ, u_e decreases with the improvement of h_d. The reason is likely that when ω increases, large head way time h_d means that the slow vehicles will bring more effect in slowing the traffic flow down. Therefore, the fact that ACC goes into effect when headways are inappropriately large will bring low efficiency in traffic running, although the traffic stability is improved. Only the tradeoff set in ACC parameters can improve traffic running efficiency and stability, and exhibit the good performance of ACC.

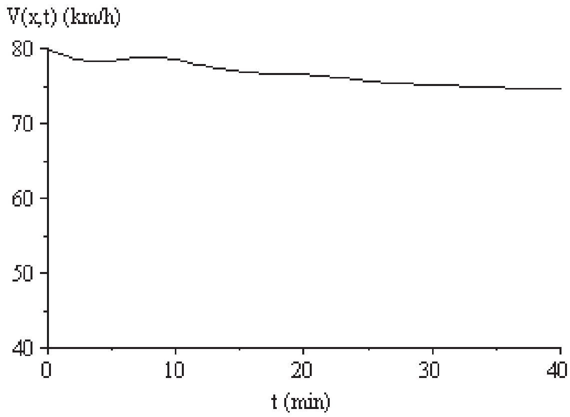

Fig. 5. Vehicular velocity at $x = 4$ Km, controlled by the model given in [1].

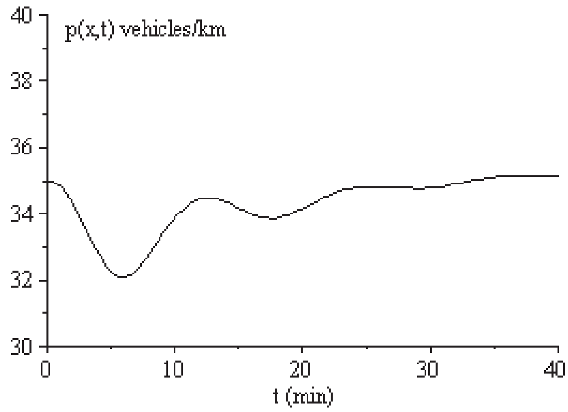

Fig. 6. Vehicular density at $x = 4$ Km, controlled by the model given in [1].

Also, in Figure 9, it can be see that with $\tau = 0.6$, in almost all of the ranges of the parameters h_d and ω, u_e is lower than that of the manually driven traffic flow, which is about 75 km/h. This means that, in terms of the efficiency of traffic flows, the ACC of the constant-headway time policy is not so good compared to manually driving instances [2]. So, replacing constant-headway time policies with other strategies for controlling traffic flows (e.g. constant-headway distance policies, which do not make the fast vehicles decelerate when their headways are too large, or variable-headway time policies, which associate the headway time of each vehicle with its instantaneous velocity, response time and headway distance, etc.) may be beneficial for preserving the running efficiency and stability of ACC traffic flows.

How does the proposed model behave when vehicle density is perturbed? To illustrate this, let $V(x,0)\rho(x,0) = c_e\rho_e$ and $\rho(x,0) = \rho_e + 5\left[\cosh^{-2}((x-5)/0.5 - \cosh^{-2}((x-7)/0.5)\right]$. The numerical simulations shown in Figures 10–13 exhibit the same characteristics shown in the case when only velocity is perturbed in (34). With these numerical simulations, we follow that: for a homogeneous steady traffic flow where $\rho_e = 35$ vehicles/km,

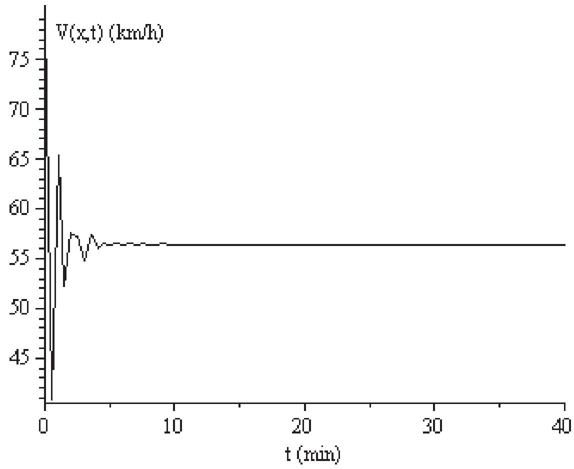

Fig. 7. vehicular velocity at $x = 4$ Km, controlled by our model.

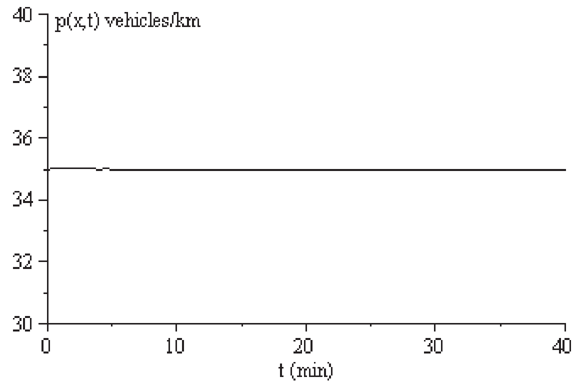

Fig. 8. Vehicular density at $x = 4$ Km, controlled by our model.

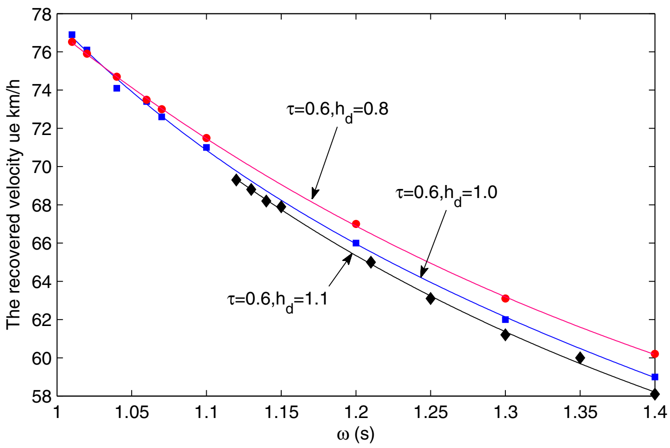

Fig. 9. With different headway time h_d, the curves of u_e vs. ω.

$c_e = 75$ km/h, when small perturbations are imposed upon c_e or p_e, if all vehicles are equipped with ACC, the traffic flow develops into a steady state more quickly, the steady vehicular density is about ρ_e, and the velocity is higher or lower than c_e, depending upon the parameter ω introduced in (26), the headway time h_d, and the perturbations. Conversely, the traffic flow constituted by

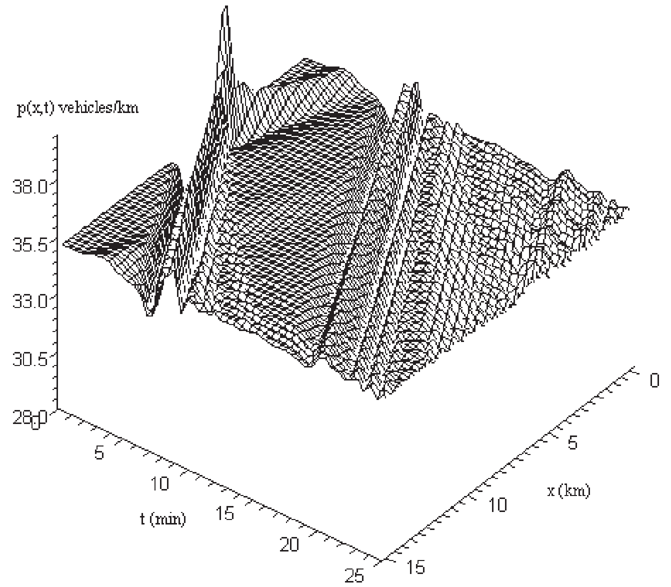

Fig. 10. The vehicular density controlled by the model given in [1], $\alpha = 60$, $\tau_0 = 300$ s.

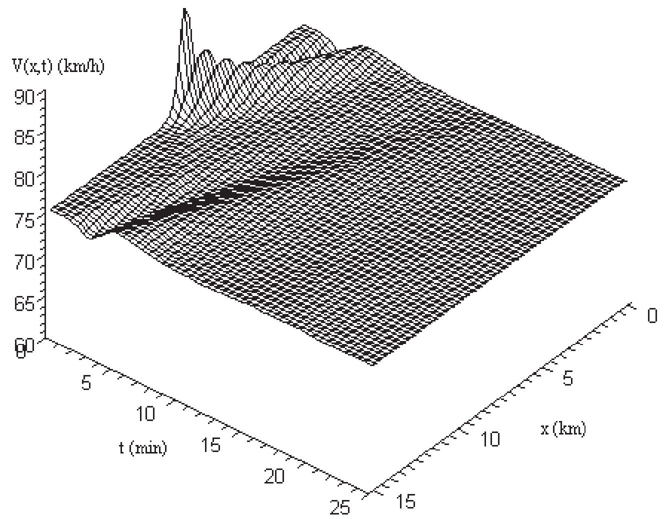

Fig. 11. The average velocity controlled by the model given in [1], $\alpha = 60$, τ_0 =300 s.

manually driven vehicles always comes back to the original steady state more slowly.

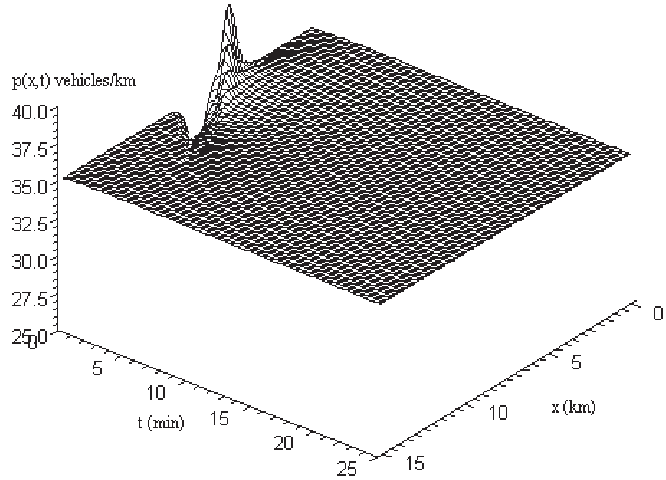

Fig. 12. The vehicular density of our model, $\tau = 60$, $h_d = 1.0$ s, $\omega = 1.2$ s and $\tau_0 = 60$ s.

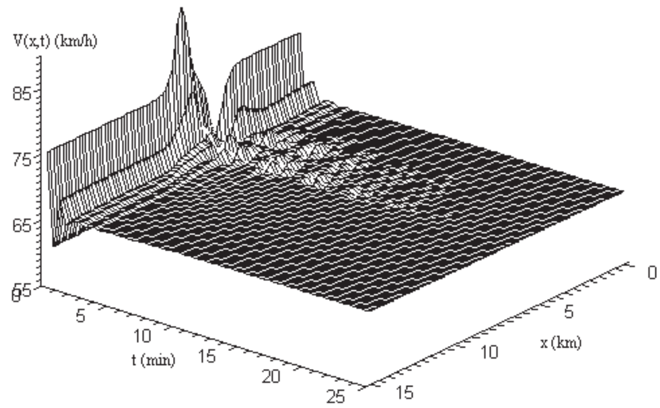

Fig. 13. The average velocity of our model, $\tau = 0.6, h_d = 1.0$ s, $\omega = 1.2$ s and $\tau_0 = 60$ s.

6 Conclusion remarks

For the traffic flow constituted by adaptive cruise control vehicles, under the hypothesis that the one-vehicle distribution function fits the Paveri-Fontana model requirement, a reduced Paveri-Fontana equation considering adaptive cruise control (ACC) effect is presented. By the maximization of the specially defined information entropy, relative to a certain reference homogeneous steady state, a set of Navier-Stokes-like equations applicable to ACC traffic flows is derived. Numerical simulations on a single circular lane indicate that, for a steady traffic flow, when small perturbations are appended to the vehicle velocity or density, less time is required for re-equilibration if all vehicles are controlled by ACC. Whether the vehicles are manually driven or controlled by ACC, the re-equilibrated steady vehicular density always equals the original value; the same is true for the re-equilibrated steady velocity for manually driven traffic flows. For ACC traffic flows, with the increase of the headway time h_d (introduced in ACC) and the pending parameter (adopted for solving the distribution function of the reference homogeneous steady state), the headway distance, at which the adjacent vehicles begin to interact, increases. This makes the relatively fast vehicles begin to decelerate when they are fairly distant from their associated preceding ones, finally leading the perturbed ACC traffic flow to stay in a steady state, whose velocity may be lower or higher than the original steady value depending upon h_d, ω, and the perturbations. Thus, an appropriate set of parameters is the foundation for good ACC performance, while an inferior set will result in low traffic running efficiency, although traffic flow stability is boosted.

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